UDC 629.7.015 : 003.62

Ref. No. : ISO/R 1151-1969 (E)

Revised

ISO

INTERNATIONAL ORGANIZATION FOR STANDARDIZATION

ISO RECOMMENDATION

R 1151

SYMBOLS FOR FLIGHT DYNAMICS

PART I

AIRCRAFT MOTION RELATIVE TO THE AIR

1st EDITION

November 1969

For each individual country the only valid standard is the national standard of that country.

Printed in Switzerland

Also issued in French and Russian. Copies to be obtained through the national standards organizations.

BRIEF HISTORY

The ISO Recommendation R 1151, *Symbols for flight dynamics – Part I : Aircraft motion relative to the air*, was drawn up by Technical Committee ISO/TC 20, *Aircraft and space vehicles*, the Secretariat of which is held by the British Standards Institution (BSI).

Work on this question led to the adoption of a Draft ISO Recommendation.

In November 1967, this Draft ISO Recommendation (No. 1484) was circulated to all the ISO Member Bodies for enquiry. It was approved, subject to a few modifications of an editorial nature, by the following Member Bodies :

Belgium	Italy	Switzerland
Canada	Netherlands	Turkey
France	New Zealand	U.A.R.
Germany	Poland	United Kingdom
India	Spain	Yugoslavia
Israel	Sweden	

Three Member Bodies opposed the approval of the Draft :

Czechoslovakia
U.S.A.
U.S.S.R.

The Draft ISO Recommendation was then submitted by correspondence to the ISO Council, which decided, in November 1969, to accept it as an ISO RECOMMENDATION.

FOREWORD

ISO Recommendation R 1151, *Symbols for flight dynamics – Part I : Aircraft motion relative to the air*, is the first in a series of ISO Recommendations, the purpose of which is to define the principal terms used in flight dynamics and to specify symbols for these terms.*

In these ISO Recommendations, the term "aircraft" denotes an aerodyne having a fore-and-aft plane of symmetry. This plane is determined by the geometrical characteristics of the aircraft. When there are more than one fore-and-aft planes of symmetry, the reference plane of symmetry is arbitrary and it is necessary to indicate the choice made.

Angles of rotation, angular velocities and moments about any axis are positive clockwise when viewed in the positive direction of the axis.

All the axis systems used are three-dimensional, orthogonal and right-handed, which implies that a clockwise (positive) rotation through $\frac{\pi}{2}$ about the x-axis brings the y-axis into the position previously occupied by the z-axis.

Numbering of sections and clauses

Each of the ISO Recommendations represents a Part of the whole study on symbols for flight dynamics.

To permit easier reference to a section or a clause from one Part to another, a decimal numbering has been adopted which begins in each Recommendation with the number of the Part it represents.

* See in Appendix X the list of Recommendations already published and the studies under way about symbols for flight dynamics.

CONTENTS

ISO Recommendation | R 1151 | November 1969

SYMBOLS FOR FLIGHT DYNAMICS

PART I

AIRCRAFT MOTION RELATIVE TO THE AIR

INTRODUCTION

This ISO Recommendation deals with the motion of the aircraft in an atmosphere at rest or in uniform motion.

To fully account for the effects of aeroelasticity and of the Earth's curvature would necessitate more detailed consideration of certain aspects of the definitions given, although these have been framed in such a way that they can be more generally interpreted. The definitions of the axes apply as they stand when the Earth's surface is treated as a plane, that is, when the Earth's radius is taken as infinite, and, in the case of the body axes, when the aircraft is treated as rigid.

1.1 AXIS SYSTEMS

No.	Term	Definition	Symbol
1.1.1	Earth-fixed axis system	A system with both origin 0_o and axes fixed with respect to the Earth, chosen to suit the problem.	x_o, y_o, z_o
1.1.2	Normal earth-fixed axis system	An earth-fixed axis system (1.1.1) in which the z_o-axis is vertically downward.	x_o, y_o, z_o but x_g, y_g, z_g is an accepted alternative.
1.1.3	Aircraft-carried earth axis system	A system in which each axis has the same direction as the corresponding earth-fixed axis, with origin 0, fixed in the aircraft, usually at the centre of gravity.	x_o, y_o, z_o
1.1.4	Aircraft-carried normal earth axis system	A system in which each axis has the same direction as the corresponding normal earth-fixed axis, with origin 0, fixed in the aircraft, usually at the centre of gravity.	x_o, y_o, z_o but x_g, y_g, z_g is an accepted alternative.
1.1.5	Body axis system	Axis system fixed in the aircraft with origin 0, usually the centre of gravity, containing the longitudinal axis, the transverse axis and the normal axis according to the following definitions :	x, y, z
	Longitudinal axis	An axis in the plane of symmetry or, if the origin lies outside this, in a parallel plane through the origin, and in some suitable forward direction.	x
	Transverse axis	An axis normal to the plane of symmetry, and positive to starboard.	y
	Normal axis	An axis in the plane of symmetry or, if the origin lies outside this, in the parallel plane through the origin, normal to the longitudinal axis, positive in the ventral sense (when viewed from the origin 0).	z
1.1.6	Air-path axis system	Axis system with aircraft fixed origin 0, usually the centre of gravity, and containing the following axes :	x_a, y_a, z_a
	x_a-axis (air-path axis)	An axis in the direction of the aircraft velocity (1.3.1)	x_a
	y_a-axis	An axis normal to the air-path axis and the z_a-axis defined below. It is positive to starboard.	y_a
	z_a-axis	An axis in the plane of symmetry, or, if the origin lies outside this, in the parallel plane through the origin and normal to the air-path axis. In normal flight conditions it is therefore ventral (when viewed from the origin 0).	z_a

1.2 ANGLES

Orientation of the aircraft velocity with respect to the body axis system (see Fig. 1).

No.	Term	Definition	Symbol
1.2.1	Angle of sideslip	The angle that the aircraft velocity (1.3.1) makes with the plane of symmetry of the aircraft. It is positive when the aircraft velocity component along the transverse axis (1.1.5) is positive. It has by convention the range $-\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2}$	β
1.2.2	Angle of attack	The angle between the longitudinal axis (1.1.5) and the projection of the aircraft velocity (1.3.1) on the plane of symmetry. It is positive when the aircraft velocity component along the normal axis (1.1.5) is positive. It has by convention the range $-\pi < \alpha \leqslant \pi$	α

Transition from the aircraft-carried normal earth axis system to the body axis system is effected by the rotations Ψ, Θ, Φ defined below, taken in that order (see Fig. 2).

NOTE. – Analogous angles can be defined with respect to any aircraft-carried earth axis system. The same symbols Ψ, Θ, Φ, with appropriate suffixes as necessary, may then be used. On the other hand, the terms azimuth angle, inclination angle and bank angle refer only to the special case where the z_o-axis is vertical.

No.	Term	Definition	Symbol
1.2.3	Azimuth angle	The rotation (positive if clockwise) about the z_o (z_g)-axis which brings the x_o (x_g)-axis into coincidence with the projection of the longitudinal axis (1.1.5) on the horizontal plane through the origin O.	Ψ
1.2.4	Inclination angle (elevation)	The rotation in a vertical plane, following the rotation Ψ (1.2.3) and which brings the displaced x_o (x_g)-axis into coincidence with the longitudinal axis (1.1.5). It is positive when the x-axis lies above the horizontal plane through the origin O. It has by convention the range $-\frac{\pi}{2} \leqslant \Theta \leqslant \frac{\pi}{2}$	Θ
1.2.5	Bank angle	The rotation (positive if clockwise) about the longitudinal axis (1.1.5) which brings the displaced y_o (y_g)-axis into its final position y from the position it reached after rotation through Ψ (1.2.3).	Φ

Transition from the aircraft-carried normal earth axis system to the air-path axis system is effected by the rotations χ_a, γ_a and μ_a defined below, taken in that order (see Fig. 3).

No.	Term	Definition	Symbol
1.2.6	Air-path azimuth angle (air-path track angle)	The rotation (positive if clockwise) about the z_o (z_g)-axis which brings the x_o (x_g)-axis into coincidence with the projection of the air-path x_a-axis (1.1.6) on the horizontal plane through the origin *0*.	χ_a
1.2.7	Air-path inclination angle (air-path climb angle)	The rotation in a vertical plane, following the rotation χ_a (1.2.6) which brings the displaced x_o (x_g)-axis into coincidence with the air-path x_a-axis (1.1.6), It is positive when the x_a-axis lies above the horizontal plane through the origin *0*. It has by convention the range $-\frac{\pi}{2} \leqslant \gamma_a \leqslant \frac{\pi}{2}$	γ_a
1.2.8	Air-path bank angle	The rotation (positive if clockwise) about the air-path x_a-axis (1.1.6) which brings the displaced y_o (y_g)-axis into its final position y_a from the position it reached after rotation through χ_a (1.2.6).	μ_a

1.3 VELOCITIES AND ANGULAR VELOCITIES

No.	Term	Definition	Symbol
1.3.1	Aircraft velocity	The velocity of the origin *0* of the body axis system (1.1.5) (usually the centre of gravity) relative to the air unaffected by the aerodynamic field of the aircraft. The corresponding scalar quantity is the airspeed.	$\vec{V}$ (V)
1.3.2	Speed of sound	The velocity of propagation of a sound wave in the ambient air unaffected by the aerodynamic field of the aircraft.	a
1.3.3	Mach number	The ratio of the airspeed (1.3.1) to the speed of sound (1.3.2). Equal to V/a.	M is recommended. However the symbols Ma and $\mathcal{M}$ may be used if otherwise there would be a possibility of confusion.

No.	Term	Definition	Symbol
1.3.4	Aircraft velocity components	The components of the velocity $\vec{V}$, for any of the axis systems used. In the axis systems 1.1.1 to 1.1.4 : component along x_o-axis component along y_o-axis component along z_o-axis In the body axis system (1.1.5) : component along the longitudinal axis component along the transverse axis component along the normal axis NOTE. – In the air-path axis system (1.1.6) the component along x_a-axis is $u_a = V$.	 u_o v_o w_o u v w For certain computations the velocity components may be written V_i where i is a dummy subscript.
1.3.5	Aircraft angular velocity	The angular velocity (corresponding scalar quantity) of the body axis system (1.1.5) relative to the Earth.	$\vec{\Omega}$ (Ω)
1.3.6	Angular velocity components Rate of roll Rate of pitch Rate of yaw	The components of the angular velocity, $\vec{\Omega}$, for any of the axis systems. In the axis systems 1.1.1 to 1.1.4 : component about the x_o-axis component about the y_o-axis component about the z_o-axis In the body axis system (1.1.5) : component about the longitudinal axis component about the transverse axis component about the normal axis	 p_o q_o r_o p q r In certain computations the angular velocity components may be written Ω_i where i is a dummy subscript.

No.	Term	Definition	Symbol
1.3.7	Normalized angular velocities	The normalized form of the components of the angular velocity (1.3.5), formed as follows : In the body axis system (1.1.5) :	
	Normalized rate of roll	$\frac{pl}{V}$	p^*
	Normalized rate of pitch	$\frac{ql}{V}$	q^*
	Normalized rate of yaw	$\frac{rl}{V}$ where l is a reference length (see 1.4.6). Similar normalized quantities can be formed for the other axis systems.	r^* Analogous quantities using a constant reference speed in place of V (1.3.1) may also be defined. These require different symbols.

1.4 AIRCRAFT INERTIA AND GEOMETRIC CHARACTERISTICS

No.	Term	Definition	Symbol
1.4.1	Aircraft mass	The current mass of the aircraft	m
1.4.2	Moments of inertia	The moments of inertia of the aircraft with respect to the body axes x, y, z (1.1.5). Moment of inertia about the longitudinal axis is $\int (y^2 + z^2) \mathrm{d}m$ Moment of inertia about the transverse axis is $\int (z^2 + x^2) \mathrm{d}m$ Moment of inertia about the normal axis is $\int (x^2 + y^2) \mathrm{d}m$	I_x I_y I_z (A, B, C are acceptable alternatives)
1.4.3	Products of inertia	The products of inertia of the aircraft with respect to the body axes x, y, z (1.1.5). These are : $\int yz \, \mathrm{d}m$ $\int zx \, \mathrm{d}m$ $\int xy \, \mathrm{d}m$	I_{yz} I_{zx} I_{xy} (D, E, F are acceptable alternatives)
1.4.4	Radius of gyration	The square root of the ratio of the moment of inertia to the aircraft mass (1.4.1) : for the longitudinal axis (1.1.5) $\sqrt{I_x/m}$ for the transverse axis (1.1.5) $\sqrt{I_y/m}$ for the normal axis (1.1.5) $\sqrt{I_z/m}$	r_x r_y r_z
1.4.5	Reference area	An area used in forming various non-dimensional quantities. For the complete aircraft the most commonly used reference area is the gross wing area (i.e. the area obtained by continuing the edges within the fuselage and the nacelles). NOTE. – Hinge moment coefficients are not usually based on this reference area.	S

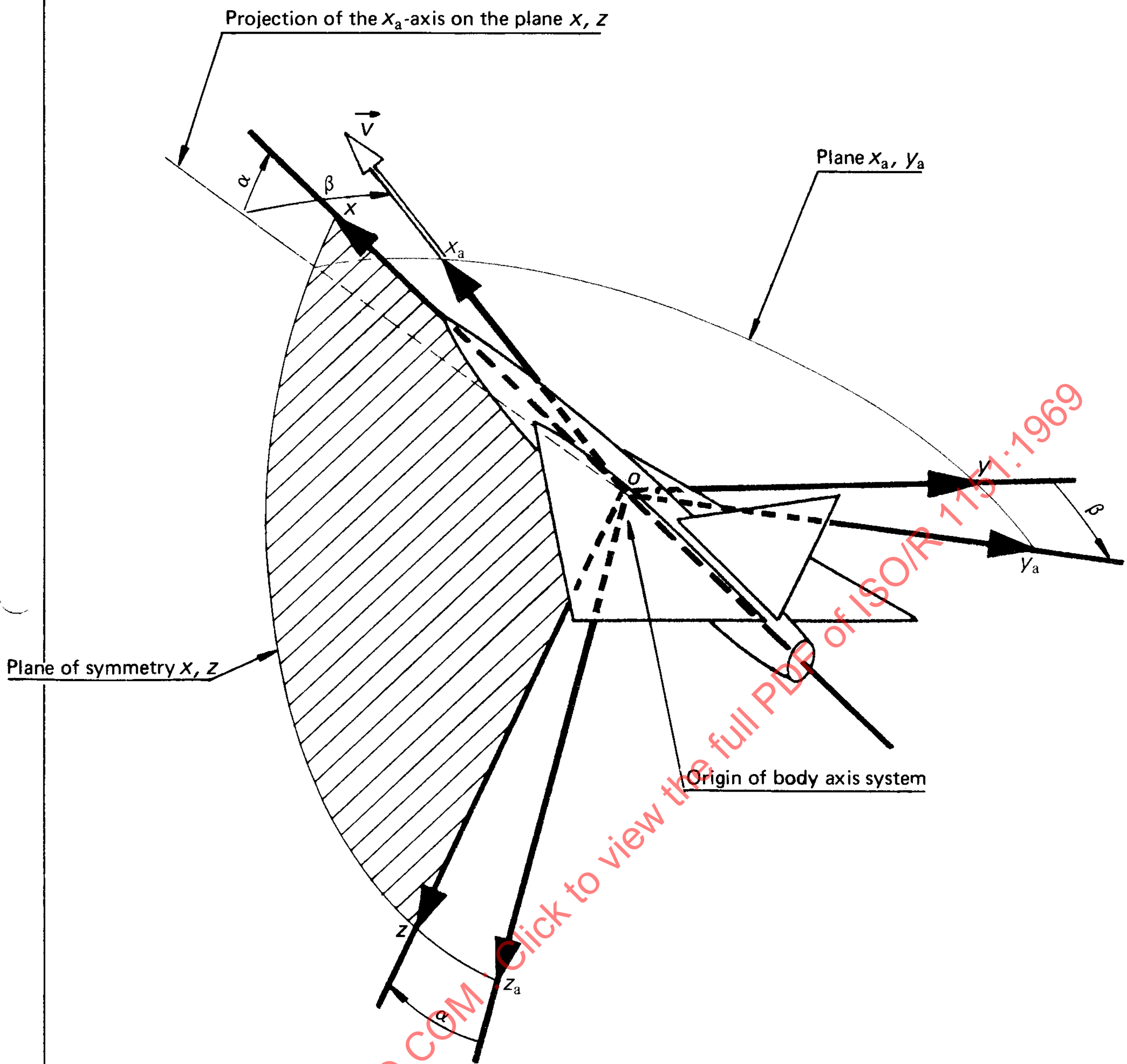
No	Term	Definition	Symbol
1.4.6	Reference length	A length used in forming non-dimensional coefficients of the aerodynamic moments and various normalized quantities. In a given document this length has a specified constant value. In the absence of a length having some aerodynamic significance the choice should correspond to an easily established geometric feature. NOTE. – Hinge moment coefficients are not usually based on this reference length.	l
1.4.7	Wing span	The distance between the two planes parallel to the plane of symmetry, tangential to the wing surface and lying wholly outside the aircraft.	b

1.5 FORCES, MOMENTS AND COEFFICIENTS

No.	Term	Definition	Symbol
1.5.1	Resultant force	The resultant vector (magnitude of the resultant vector) of the system of forces acting on the aircraft including the aerodynamic forces and the propulsion forces, but excluding the gravitational, inertial and reaction forces due to contact with the Earth's surface. NOTE. – In the special cases where only the airframe aerodynamic forces or the propulsive forces are considered, a distinguishing symbol is necessary (see 1.6*).	$\vec{R}$ (R)
1.5.2	Components of the resultant force	The components of the resultant force vector, $\vec{R}$. In the body axis system (1.1.5) : component along the longitudinal axis component along the transverse axis component along the normal axis In the air-path axis system (1.1.6) : component along the x_a-axis component along the y_a-axis component along the z_a-axis	 X Y Z X_a Y_a Z_a
1.5.3	Force coefficients	Non-dimensional coefficients of the resultant force (1.5.1), formed as follows : In the body axis system (1.1.5) : X force coefficient is $X/\frac{1}{2}\rho V^2 S$ Y force coefficient is $Y/\frac{1}{2}\rho V^2 S$ Z force coefficient is $Z/\frac{1}{2}\rho V^2 S$ In the air-path axis system (1.1.6) : X_a force coefficient is $X_a/\frac{1}{2}\rho V^2 S$ Y_a force coefficient is $Y_a/\frac{1}{2}\rho V^2 S$ Z_a force coefficient is $Z_a/\frac{1}{2}\rho V^2 S$ where ρ is the density of the air unaffected by the aerodynamic field of the aircraft. NOTE. – These definitions are not the ones usually used in helicopter studies.	 C_X C_Y C_Z C_{Xa} C_{Ya} C_{Za}

* 1.6 – Thrust, (airframe) aerodynamic force and their components. At present at the stage of draft proposal.

No.	Term	Definition	Symbol
1.5.4	Resultant moment	The resultant moment of the system of forces forming the resultant force of 1.5.1 about a reference point, usually the centre of gravity.	
1.5.5	Components of the resultant moment	The components of the resultant moment.	
		In the body axis system (1.1.5) :	
	Rolling moment	component about the longitudinal axis	L
	Pitching moment	component about the transverse axis	M
	Yawing moment	component about the normal axis	N
		In the air-path axis system (1.1.6) :	
		component about the x_a-axis	L_a
		component about the y_a-axis	M_a
		component about the z_a-axis	N_a
1.5.6	Moment coefficients	Non-dimensional coefficients of the resultant moment (1.5.4) formed as follows :	
		In the body axis system (1.1.5) :	
	Rolling moment coefficient	$L/\frac{1}{2}\rho V^2 Sl$	C_l
	Pitching moment coefficient	$M/\frac{1}{2}\rho V^2 Sl$	C_m
	Yawing moment coefficient	$N/\frac{1}{2}\rho V^2 Sl$	C_n
		In the air-path axis system (1.1.6) :	
		$L_a/\frac{1}{2}\rho V^2 Sl$	C_{la}
		$M_a/\frac{1}{2}\rho V^2 Sl$	C_{ma}
		$N_a/\frac{1}{2}\rho V^2 Sl$	C_{na}
		where ρ is the density of the air unaffected by the aerodynamic field of the aircraft.	
		NOTE. – These definitions are not the ones usually used in helicopter studies.	

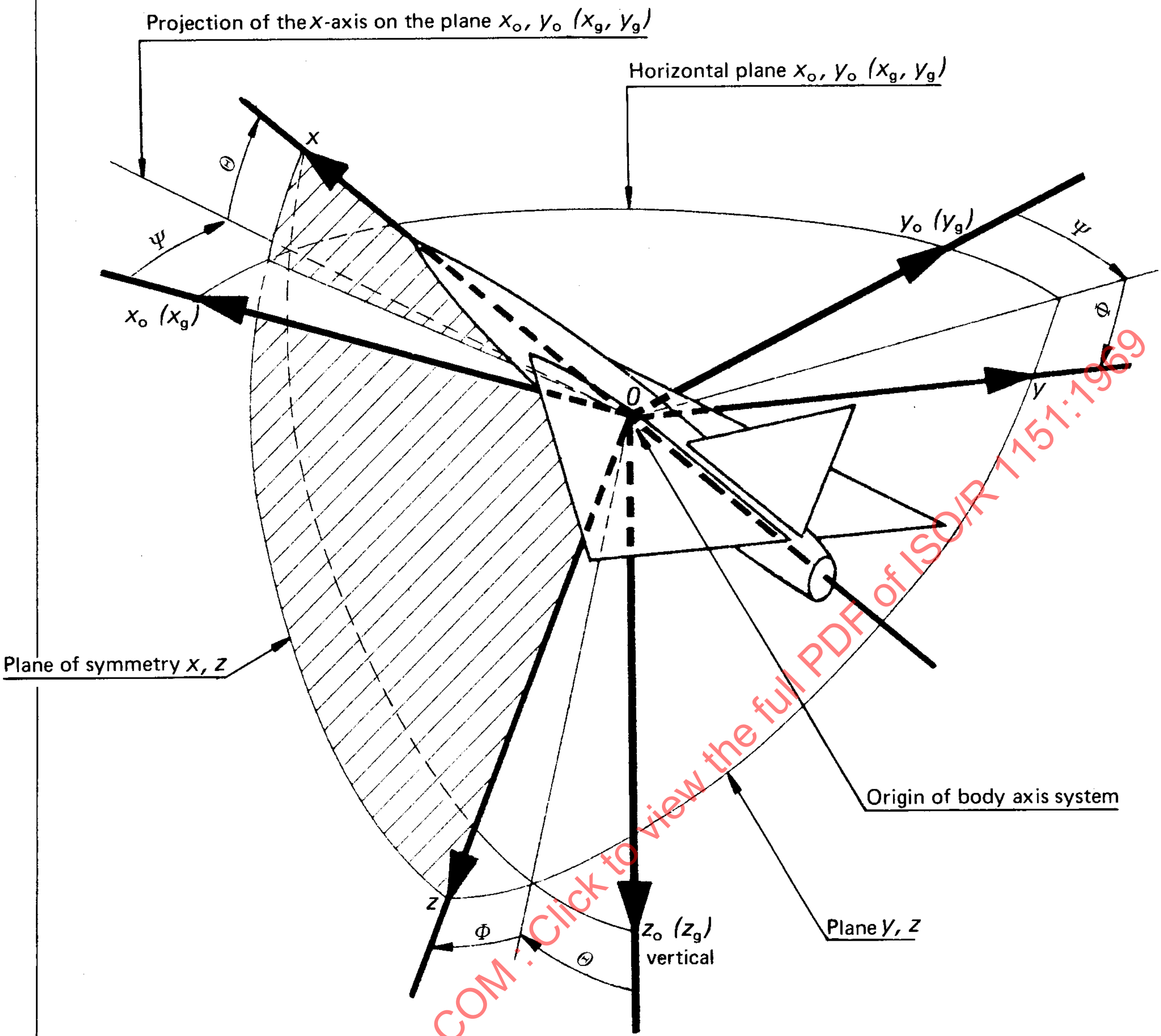


IN BLACK : Body axis system

IN BLUE : Air-path axis system

NOTE. – The angles shown are positive. When O is not in the plane of symmetry, the x, z-plane is parallel to it and the angles are unchanged.

FIG. 1 – **Orientation of the aircraft velocity with respect to the body axis system**



IN RED : Aircraft-carried normal earth axis system

IN BLACK : Body axis system

NOTE. – The angles shown are positive. When *O* is not in the plane of symmetry, the *x*, *z*-plane is parallel to it and the angles are unchanged.

FIG. 2 – **Orientation of the body axis system relative to the aircraft-carried normal earth axis system**